Gravity, Orbital Motion, & Relativity
Early Astronomy
Early Times

• As far as we know, humans have always been interested in the motions of objects in the sky.

• Not only did early humans navigate by means of the sky, but the motions of objects in the sky predicted the changing of the seasons, etc.

• There were many early attempts both to describe and explain the motions of stars and planets in the sky.

• All were unsatisfactory, for one reason or another.
The Earth-Centered Universe

- A geocentric (Earth-centered) solar system is often credited to Ptolemy, an Alexandrian Greek, although the idea is very old.

- Ptolemy’s solar system could be made to fit the observational data pretty well, but only by becoming very complicated.
Center of epicycle moves counterclockwise on deferent and epicycle moves counterclockwise. Epicycle speed is uniform with respect to equant. The combined motion is shown at right.

Deferent motion is in direction of point 1 to 7 but planet's epicycle carries it on cycloid path (points 1 through 7) so that from points 3 through 5 the planet moves backward (retrograde).
Copernicus’ Solar System

- The Polish cleric **Copernicus** proposed a **heliocentric** (Sun centered) solar system in the 1500’s.
Objections to Copernicus

How could Earth be moving at enormous speeds when we don’t feel it?
- (Copernicus didn’t know about **inertia**.)

Why can’t we detect Earth’s motion against the background stars (stellar parallax)?

Copernicus’ model did **not** fit the observational data very well.
• Galileo Galilei - February 15, 1564 – January 8, 1642

• **Galileo** became convinced that Copernicus was correct by **observations** of the Sun, Venus, and the moons of Jupiter using the newly-invented telescope.

• Perhaps Galileo was motivated to understand **inertia** by his desire to understand and defend Copernicus’ ideas.
Orbital Motion
Tycho and Kepler

• In the late 1500’s, a Danish nobleman named Tycho Brahe set out to make the most accurate measurements of planetary motions to date, in order to validate his own ideas of planetary motion.

• Tycho’s data was successfully interpreted by the German mathematician and scientist Johannes Kepler in the early 1600’s.
Tycho Brahe

- **Tycho Brahe** (14 December 1546 – 24 October 1601)

- Danish nobleman known for his accurate and comprehensive astronomical and planetary observations
Johannes Kepler

- **Johannes Kepler**: (December 27, 1571 – November 15, 1630)

- German mathematician, astronomer and astrologer.

- Based upon observation made by Tycho Brahe, developed “Three Laws of Planetary Motion”

- **Kepler** determined that the orbits of the planets were not perfect circles, but **ellipses**, with the Sun at one focus.
Kepler’s Three Laws of Planetary Motion

1st Law: “Law of Ellipses”

2nd Law: “Law of Equal Areas”

“Law of Ellipses”

The orbit of every planet is an ellipse with the Sun at a focus.
“Law of Equal Areas”

A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.

Kepler determined that a planet moves faster when near the Sun, and slower when far from the Sun.
“Law of Harmonies”

The square of the orbital period (time for one orbit) of a planet is directly proportional to the cube of the semi-major axis (average distance from the Sun) of its orbit.

\[ P_a^2 / P_b^2 = a_a^3 / a_b^3 \]

And

\[ \frac{P_{\text{planet}}^2}{a_{\text{planet}}^3} = \frac{P_{\text{earth}}^2}{a_{\text{earth}}^3} \]
Why?

Kepler’s Laws provided a complete *kinematical* description of planetary motion (including the motion of planetary satellites, like the Moon) - but *why* did the planets move like that?
Gravity
Sir Issac Newton

- Sir Issac Newton - Dec. 25 1642 – March 2, 1727

- Went to the family farm in the country to escape the Black Plague in 1665 and noticed an apple falling from a tree

- Led him to wonder about how high gravity reached

- Knew of Kepler’s three laws of planetary motion
The Apple & the Moon

• Isaac Newton realized that the motion of a falling apple and the motion of the Moon were both actually the **same motion**, caused by the **same force** - the **gravitational force**.

• Newton’s idea was that gravity was a **universal** force acting between **any two objects**.

• All things in the universe are attracted to all other things with mass.
At the Earth’s Surface

- Newton knew that the gravitational force on the apple equals the apple’s weight, $mg$, where $g = 9.8 \text{ m/s}^2$. 

\[ W = mg \]
Newton reasoned that the centripetal force on the moon was also supplied by the Earth’s gravitational force.
Weight of the Moon

• Newton’s calculations showed that the centripetal force needed for the Moon’s motion was about $1/3600^{th}$ of $Mg$, however, where $M$ is the mass of the Moon.

• Newton knew, though, that the Moon was about $60$ times farther from the center of the Earth than the apple.

• And $60^2 = 3600$
Universal Gravitation (Inverse Square Law)

- From this, Newton reasoned that the strength of the gravitational force is **not constant**, in fact, the magnitude of the force is **inversely proportional to the square of the distance** between the objects.

- Newton concluded that the gravitational force is:
  - **Directly proportional** to the masses of both objects.
  - **Inversely proportional** to the distance between the objects.
Mass: Direct Relationship
Distance: Inverse Square Relationship
Newton’s Law of Universal Gravitation

This equation relates mass and distance to force

\[ F = G \frac{m_1 m_2}{r^2}, \]

where:
- \( F \) is the force between the masses,
- \( G \) is the gravitational constant,
- \( m_1 \) is the first mass,
- \( m_2 \) is the second mass, and
- \( r \) is the distance between the masses.

\[ G = 6.67428 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} = 6.67428 \times 10^{-11} \text{ N (m/kg)}^2 \]
Ok its more like this

Newton's law of universal gravitation is not as romantic

\[ F_w = G \frac{m_E m}{d^2} \]
The value of \( G \) was not experimentally determined until nearly a century later (1798) by Lord Henry Cavendish using a torsion balance.

Cavendish's apparatus involved a light, rigid rod about 2-feet long. Two small lead spheres were attached to the ends of the rod and the rod was suspended by a thin wire. When the rod becomes twisted, the torsion of the wire begins to exert a torsional force that is proportional to the angle of rotation of the rod. The more twist of the wire, the more the system pushes *backwards* to restore itself towards the original position.
An Inverse-Square Force

\[ F_{\text{grav}} = \frac{GMm}{r^2} \]

You get 4 times the force at half the distance.

You get 1/4 the force at twice the distance.

You get 1/16 the force at 4 times the distance.

You get 1/9 the force at 3 times the distance.
Experimental Evidence

- The Law of Universal Gravitation allowed extremely accurate predictions of planetary orbits.
- Cavendish measured gravitational forces between human-scale objects before 1800. His experiments were later simplified and improved by von Jolly.
Action at a Distance

• In Newton’s time, there was much discussion about HOW gravity worked - how does the Sun, for instance, reach across empty space, with no actual contact at all, to exert a force on the Earth?

• This spooky notion was called “action at a distance.”
The Gravitational Field

- During the 19th century, the notion of the “field” entered physics (via Michael Faraday).

- Objects with mass create an **invisible disturbance in the space around them** that is felt by other massive objects - this is a **gravitational field**.

- So, since the Sun is very massive, it creates an intense gravitational field around it, and the **Earth responds to the field**. No more “action at a distance.”
Gravitational Field Strength

• To measure the strength of the gravitational field at any point, measure the gravitational force, $F$, exerted on any “test mass”, $m$.

• **Gravitational Field Strength, $g = F/m$**

• Near the surface of the Earth, $g = F/m = 9.8 \text{ N/kg} = 9.8 \text{ m/s}^2$.

• In general, $g = \frac{GM}{r^2}$, where $M$ is the mass of the object creating the field, $r$ is the distance from the object’s center, and $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$. 
Gravitational Force

• If \( g \) is the strength of the gravitational field at some point, then the gravitational force on an object of mass \( m \) at that point is \( F_{\text{grav}} = mg \).

• If \( g \) is the gravitational field strength at some point (in N/kg), then the free fall acceleration at that point is also \( g \) (in m/s\(^2\)).
Gravitational Field Inside a Planet

- If you are located a distance \( r \) from the center of a planet:
  - all of the planet’s mass inside a sphere of radius \( r \) pulls you *toward the center* of the planet.
  - All of the planet’s mass outside a sphere of radius \( r \) exerts *no* net gravitational force on you.
Gravitational Field Inside a Planet

- The blue-shaded part of the planet pulls you toward point C.
- The grey-shaded part of the planet does not pull you at all.
Gravitational Field Inside a Planet

- Half way to the center of the planet, $g$ has one-half of its surface value.
- At the center of the planet, $g = 0 \text{ N/kg.}$
The Earth

Earth’s mass = \(6.0 \times 10^{24}\) kg
Earth's radius = \(6.4 \times 10^6\) m
Relationship between Mass and Weight

Your weight (on Earth) is the force felt between you and earth.

There are two ways to calculate the force of weight:
1. Using the mass of two objects, distance between, and G.

\[ F_w = G \frac{m_E \cdot m}{d^2} \]

Bob : \( m_1 = 89 \text{ kg} \)

Earth : \( m_2 = 6.0 \times 10^{24} \)

Earth's radius = \( 6.4 \times 10^6 \text{ m} \)
Relationship between Mass and Weight

2. Using the mass of one if $g$ (the acceleration due to gravity) is known

$$F_w = m \cdot g$$

Bob: $m_1 = 89$ kg

$g = 9.80 \text{ m/s}^2$ on Earth
Relationship between Mass and Weight

- $G$ is not $g$

- $g$: acceleration due to gravity (depends on location)
  \[ g = 9.80 \text{ m/s}^2 \text{ on Earth} \]

- $G$: Universal gravitational constant (everywhere)
  \[ G = 6.67 \times 10^{-11} \frac{\text{N}(\text{m}^2)}{\text{kg}^2} \]

\[
F_w = m \cdot g
\]

\[
F_w = G \frac{m_E \cdot m}{d^2}
\]
Acceleration of an Object Due to the Earth

\[ F_{\text{grav}} = F_{\text{grav}} \]
\[ m_1 a = G m_1 M_E / r^2 \]
\[ a = G M_E / r^2 \]
\[ a = \left[ (6.67 \times 10^{-11} \text{Nm}^2/\text{kg}^2) \times (5.97 \times 10^{24} \text{kg}) \right] / (6.4 \times 10^6 \text{m})^2 \]
\[ a = 9.8 \text{ m/s}^2 \]
Acceleration of the Earth Due to an Object

\[ F_{\text{grav}} = F_{\text{grav}} \]
\[ M_e a = G m_1 M_E / r^2 \]
\[ a = G m_1 / r^2 \]

Let \( m_1 = 100 \text{ kg} = (220 \text{ lbs}) \)

\[ a = \frac{(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2) \times (100 \text{ kg})}{(6.4 \times 10^6 \text{ m})^2} \]

\[ a = 1.6 \times 10^{-22} \text{ m/s}^2 \]

Too small to observe...

\[ .00000000000000000000000016 \text{ m/s}^2 \]
Earth’s Tides

- There are 2 high tides and 2 low tides per day.
- The tides follow the Moon.
Why *Two* Tides?

- Tides are caused by the stretching of a planet.
- Stretching is caused by a difference in forces on the two sides of an object.
- Since gravitational force depends on distance, there is more gravitational force on the side of Earth closest to the Moon and less gravitational force on the side of Earth farther from the Moon.
Why **Two** Tides?

- Remember that

\[ F_{grav} = G \frac{Mm}{r^2} \]

**Less gravitational force on the far side.**

**More gravitational force on the near side.**
Why the **Moon**?

- The Sun’s gravitational pull on Earth is much larger than the Moon’s gravitational pull on Earth. So why do the tides follow the Moon and not the Sun?

- Since the Sun is much farther from Earth than the Moon, the difference in distance across Earth is much less significant for the Sun than the Moon, therefore the difference in gravitational force on the two sides of Earth is less for the Sun than for the Moon (even though Sun’s force on Earth is more).

- The Sun does have a small effect on Earth’s tides, but the major effect is due to the Moon.
Circular Motion
Circular Motion Terms

**Axis** - is the straight line around which rotation takes place.

**Two types of circular motion**

**Rotation** - also called spin; when an object turns around an internal axis.

**Revolution** - When an object turns around an external axis.
Centripetal Force ($F_c$)

Any force directed toward the center of the circle.

What could cause an object to stay in orbit giving a centripetal force

- A rope: $F_c = F_T$ tension in the rope
- Gravity: $F_c = F_g$ gravity
- Friction: $F_c = F_f$ friction
What would happen if an object were released from its orbit?

Draw the direction it would travel if released at 1 and 2.
What would happen if an object were released from its orbit?

Draw the direction it would travel if released at 1 and 2

The object would travel tangent to the circle in a straight line
Equations for Centripetal Force ($F_c$)

\[ F_c = ma_c \]

\[ a_c = \frac{v^2}{r} \]

\[ F_c = \frac{mv^2}{r} \]

\[ v = \frac{2\pi r}{T} \]

\[ T = \frac{1}{f} \]

$T =$ period (s) seconds per rotation

$f =$ frequency (Hz) rotations per
**Centrifugal Force** - a fictitious outward force, caused by an object's inertia, felt when an object follows a circular path.

It feels like you would move straight out away from the circle.
Satellite Motion

• A satellite is often thought of as being a projectile which is orbiting the Earth.

• But how can a projectile orbit the Earth?

• Doesn't a projectile accelerate towards the Earth under the influence of gravity?

• And as such, wouldn't any projectile ultimately fall towards the Earth and collide with the Earth, thus ceasing its orbit?
Satellite Motion

- First, an orbiting satellite is a projectile in the sense that the only force acting upon an orbiting satellite is the force of gravity.
- Most Earth-orbiting satellites are orbiting at a distance high above the Earth such that their motion is unaffected by forces of air resistance.
- Indeed, a satellite is a projectile.
Satellite Motion

- Second, a satellite is acted upon by the force of gravity and this force *does* accelerate it towards the Earth.

- In the absence of gravity a satellite would move in a straight line path tangent to the Earth.

- In the absence of any forces whatsoever, an object in motion (such as a satellite) would continue in motion with the same speed and in the same direction.

- This is the **law of inertia**. The force of gravity acts upon a high speed satellite to deviate its trajectory from a straight-line inertial path.

- Indeed, a satellite is accelerating towards the Earth due to the force of gravity.
Satellite Motion

• Finally, a satellite does fall towards the Earth; only it never falls into the Earth.

• To understand this concept, we have to remind ourselves of the fact that the Earth is round; that is the Earth curves.

• In fact, scientists know that on average, the Earth curves approximately 5 meters downward for every 8000 meters along its horizon.

• If you were to look out horizontally along the horizon of the Earth for 8000 meters, you would observe that the Earth curves downwards below this straight-line path a distance of 5 meters.
Satellite Motion

• In order for a satellite to successfully orbit the Earth, it must travel a horizontal distance of 8000 meters before falling a vertical distance of 5 meters.

• A horizontally launched projectile falls a vertical distance of 5 meters in its first second of motion.

• To avoid hitting the Earth, an orbiting projectile must be launched with a horizontal speed of 8000 m/s. When launched at this speed, the projectile will fall towards the Earth with a trajectory which matches the curvature of the Earth.

• As such, the projectile will fall around the Earth, always accelerating towards it under the influence of gravity, yet never colliding into it since the Earth is constantly curving at the same rate.

• Such a projectile is an orbiting satellite.
Launch Speed less than 8000 m/s
Projectile falls to Earth

Launch Speed equal to 8000 m/s
Projectile orbits Earth - Circular Path

Launch Speed greater than 8000 m/s
Projectile orbits Earth - Elliptical Path
Satellite Math

\[ F_{net} = \left( M_{sat} \cdot v^2 \right) / R \]

\[ a = \frac{G \cdot M_{central}}{R^2} \]

\[ v = \sqrt{\frac{G \cdot M_{central}}{R}} \]

\[ \frac{T^2}{R^3} = \frac{4 \cdot \pi^2}{G \cdot M_{central}} \]
Escape Velocity

\[ v_e = \sqrt{\frac{2GM}{r}} \]

\[ m = \text{mass of central body} \]
Relativity

SPACE

TIME

ENERGY

MATTER

1905 SPECIAL THEORY OF RELATIVITY

1905 ENERGY-MASS EQUIVALENCE

1915 GENERAL THEORY OF RELATIVITY
Albert Einstein

- Albert Einstein - March 14, 1879 - April 18, 1955
- Special Relativity
- General Relativity
Events & Inertial Reference Frames

- An event, is a physical happening that occurs at certain place and time

- Observers of an event use a reference frame
  1. Consists of a set of x, y, z axes (coordinate system) and a clock
  2. Coordinate systems are used to establish where, and clocks specify when

- Theory of Relativity deals with a “special” kind of reference frame called and inertial reference frame

  - Is a frame in which Newton’s law of inertia is valid – if net force acting on body = 0, then body remains at rest or moves at a constant velocity
General Relativity (1916)

- Gravity is a geometric property of space and time (space-time).

Einstein approaches the concept of gravity, not as force so much as a distortion of the space time continuum.

- Essentially, massive objects “curve” space-time (4D: 3 Spatial & 1 Temporal Dimensions) to create gravitational fields (“gravitational potential energy wells”).
\[ R_{\eta\eta} = -\frac{2a^2}{\delta\psi} \frac{\partial^2 \psi}{\partial \theta^2} \cot \theta + \frac{2ac}{\delta\psi} \frac{\partial^2 \psi}{\partial \eta^2} \cot \theta + \frac{a}{\delta} \frac{\partial^2 \psi}{\partial \theta \partial \eta} \cot \theta - \frac{\partial a}{\partial \eta} \frac{\partial \psi}{\delta} \cot \theta - \frac{2a^2}{\delta^2} \frac{\partial^2 \psi}{\partial \eta^2} \]
Distortion of the Space-Time Continuum
Black Holes

- When a very massive star gets old and runs out of fusionable material, gravitational forces may cause it to collapse to a mathematical point - a singularity. All normal matter is crushed out of existence. This is a black hole.

- If depression is deep enough (gravity strong enough by Newton’s equation), light cannot escape from the depression, creating a black hole.
Black Hole Gravitational Force

Gravitational Force (arbitrary units)

Distance from center of black hole (in star radii)

Black Hole Gravitational Force

Normal Star Gravitational Force

Gravitational force outside R = 1 is the same for both!
Black Hole Gravitational Force

• The black hole’s gravity is the same as the original star’s at distances greater than the star’s original radius.

• Black hole’s don’t magically “suck things in.”

• The black hole’s gravity is intense because you can get really, really close to it!
Wormhole

- Hypothetical topological feature of space time that would be, fundamentally, a "shortcut" through space time.

- much like a tunnel with two ends each in separate points in space time
But Before General Relativity…Special Relativity (1905), which allows us to understand the dynamics of objects moving at speeds comparable to the speed of light.
Two Postulates of Special Relativity

1. **The Relativity Postulate** (Equivalence of Physical Laws): The laws of physics are the same in all “inertial reference frames” (systems moving at constant speeds relative to each other).

2. **The Speed of Light Postulate** (Constancy of the Speed of Light): Essentially, whenever you measure the speed of light, you get \( c \) (\( c = 186,000 \text{ mi/s} = 3.0 \times 10^8 \text{ m/s} \)) regardless of the motion of the source or observer.
The Relativity of Time and Time Dilation

Time is not absolute- If the speed of light stays the same then what is going on? Something else has to change. That "something" is time.

As odd as it seems time is not constant. More accurately, space-time is not constant. It can be changed, bent and twisted. The faster we go the more time slows down ("moving clocks run slow"). This is only noticeable, normally, at very high speeds such as those approaching the speed of light, 300,000 km per second, which is approximately 7 times around the Earth in a second.

Moving clocks run slower than stationary clocks.

In a 4D universe, the more you move through 3D the less you move in the fourth (time)
Time Dilation

Hypothetical Example
(Not true values)

\[ v = 100v_1 \]
\[ t = t_0 \]

1.0 sec
(stopwatch)

\[ v = zv_1 \]
\[ t = t_0 \]

1.6 s
(stopwatch)

\[ v = v_1 \]
\[ t = t_0 \]

2.0 sec
(stopwatch)
Time Dilation = to “dilate” means to “expand”

\[ t = \frac{t_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \]

- \( t_0 \) = proper time (as measured by observer)
- \( t \) = dilated time measured by observer

If \( v = 0 \), \( t = t_0 \)

So, \( c \) is the universal speed limit
Relativity of Length and Length Contraction

- Length is not absolute

Moving objects “contract” in the direction of their motion “contract” is to “shorten”

Length Contraction – shortening of the distance between two points
Length Contraction

\[ l = l_0 \sqrt{1 - \frac{v^2}{c^2}} \]

- \( l_0 \) = proper length (as measured by observer)
- \( l \) = contracted length
- If \( v = 0 \), \( l = l_0 \)
- If \( v = c \), \( l = 0 \)
Relativistic Mass

- Mass is relative

Moving objects increase in mass

When a body is moving, we find that its force–acceleration relationship is no longer constant, but depends on two quantities: its speed, and the angle between its direction of motion and the applied force.
Relativistic Mass

\[ m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \]

- \( m_0 \) = proper mass
- \( m \) = relativistic mass measured by observer
- If \( v = 0 \), \( m = m_0 \)
- So, objects with mass cannot travel at \( c \)
Relativistic Energy

- Mass and Energy are equivalent (loss or gain in either equal)
- Total Energy of moving object is related to mass by
  \[ E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \]
  if at rest \( v=0 \) then
  \[ E_o = mc^2 \]

- \( E_o = \) Rest Energy
- \( m = \) Rest Mass
Evidence

- **Time Dilation:** Atomic clocks in airplanes

- **Length Contraction:** Hard to observe but can be done with space based satellites

- **Relativistic Mass:** Particle accelerators

- **Relativistic Energy:** Nuclear power and atom bombs
Moral Relativity

Related to moral relativism, it states that ethics become subjective only when you approach the speed of light. That is, it's OK to be self-serving, steal, and murder as long as you're going really, really fast.

(Note: This is why rap sounds better on the highway at 90 mph)
Twin’s Paradox

Did the space bound twin age Or did the Earth-bound twin do all the aging??
Ladder Paradox

Is this why parking spaces look bigger than they are when you try and parallel park?